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Aerodynamic admittance functions and buffeting forces for bridges via indicial functions

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Abstract

Buffeting forces on bridge decks are commonly modelled by Sears' function. However, it is well known that Sears' function is reliable only for very streamlined bridge deck sections and that a complete model would require a suitable formulation of buffeting forces in time domain. In this paper, self-excited and buffeting loads are modelled by means of indicial functions. Corresponding aerodynamic admittance functions are numerically evaluated for rectangular sections and compared with experimental and analytical results. A complete time-domain model for cross-sections including vertical turbulence is presented. Numerical simulations are performed on a sample rectangular section. Comparison with experimental results and relevant flutter analyses are also discussed.

Keywords: Bridges; Time-domain forces; Indicial functions; Admittance; Buffeting; Flutter

1. Introduction

Time-domain simulations are a powerful tool to evaluate the dynamic behaviour of bridges in atmospheric flow. In fact, structural nonlinearity, which can be prominent in the case of long or super-long spans, can be accounted for in the analysis, together with turbulence effects. External wind loads are commonly modelled as a combination of three actions: (i) a quasi-steady component related to mean incoming wind, (ii) a self-excited part linearly dependent on structural motion, and (iii) a buffeting fraction due to turbulent flow. These aspects are included here in a unique time-domain formulation, neglecting their possible interaction. Attention is focused on load models for buffeting.

1.1. Background: liaison with thin airfoil

Load models commonly used to pattern the wind action on bridges follow the pioneering work by Davenport (1962) on the response to gusty winds and by Scanlan and Tomko (1971) who worked on the modelling of flutter and buffeting forces.

Such load models are built up by recalling the strong parallelism between thin airfoils and streamlined bridge decks. A *thin airfoil* is assumed to be the theoretical reference section and referred to also as a *flat plate* (*FP*). The main hypotheses on which thin airfoil theory is based, namely the existence of potential flow, the absence of separation of

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shear layers and full gust coherence, do not apply strictly to bodies with different geometrical shapes. As a matter of fact, no theoretical formulation is available to treat aeroelastic problems for bluff bodies like bridges. However, tools of thin airfoil theory can furnish straightforward information also on bridge deck behaviour. Therefore, formulation of external loads follows the guidelines of aerodynamic theory, but adapts the theory itself to the case of each bridge with the introduction of proper experimental data.

The buffeting response of a bridge deck predicted with Davenport's spectral approach is mainly based on modelling of gust loading as a stationary random process, on the quasi-steady assumption and on the strip assumption, as suggested by aerodynamicists Sears (1941) and Liepmann (1955). In particular, it is assumed that (i) buffeting action on each section is independent from the action on the contiguous sections ('*strip assumption*'), (ii) the buffeting force applying on a section is induced by a gust persisting for an infinitely long time ('*quasi-steady assumption*'), and (iii) the effect of different wavelengths can be accounted for by means of *aerodynamic admittance functions*. The buffeting response is computed mode-by-mode, neglecting aerodynamic coupling, while a joint admittance function describes the distribution of buffeting forces along the bridge span.

The approach suggested by Scanlan and co-workers, furthermore, superimposes buffeting forces due to turbulent wind components and self-excited forces due to flow–structure interaction. Both buffeting and self-excited forces are included in a time-domain framework, common also to thin airfoil aerodynamics and characterized by frequency-dependent functions. In particular, buffeting forces are expressed by means of aerodynamic admittance functions, while self-excited forces are modelled via *aeroelastic* or *flutter derivatives*. These frequency-dependent functions are generally available in terms of discrete-frequency parameters measured in wind tunnel tests.

The quantities valid for a thin airfoil and theoretically corresponding to aerodynamic admittance functions and flutter derivatives, are known, respectively, as *Sears' function* and *Theodorsen's circulation function*. In particular, Sears' function allows one to calculate lift due to a sinusoidal vertical gust encountering the airfoil (Sears, 1941), whereas Theodorsen's function is employed to describe self-excited lift developing on an airfoil in sinusoidal oscillation (Theodorsen, 1935).

A time-domain representation of both buffeting and self-excited actions, without explicit dependence on frequency, can be provided if appropriate functions are defined. According to thin airfoil theory, such functions describe the time dependence of sectional forces due to elementary gusts or structural motion. The total resulting action can be calculated via convolutions, accounting for arbitrary gusts and motions. In thin airfoil theory, *Küssner's (1936) function* and *Wagner's (1925) function* provide, respectively, lift due to a sharp-edged gust investing the airfoil and self-excited lift induced by an elementary step change in the angle of attack, that is in the relative placement of the section with respect to the incident flow. Such functions are usually referred to as *indicial functions*. Corresponding indicial functions for bridges can be analogously defined as Küssner- and Wagner-like functions, respectively, for buffeting and self-excited forces. Indicial functions describing self-excited forces are calculated from flutter derivatives, rather than directly identified in wind tunnel tests. Indicial functions for buffeting cannot be found in the literature. Commonly, a frequency-based approach is adopted and Sears' function is used to calculate buffeting response.

1.2. Literature survey: aerodynamic admittance functions and buffeting forces

After Davenport's and Scanlan's early work, several efforts have been carried out to remove the main assumptions concerning buffeting response and characterize aerodynamic admittances.

For example, Scanlan (1984) discusses the effect of turbulence action within a load model which includes self-excited forces modelled by means of indicial functions. Buffeting loads are defined via admittance functions and the analysis is basically performed in the frequency domain.

The first time-domain formulation of wind action including buffeting and self-excited forces both modelled via indicial functions is due to Scanlan (1993). In this work, different types of indicial functions are proposed to account for unsteady gusty and self-excited excitations.

Moreover, several discussions on the existence of phenomenological relationships between buffeting and self-excited parameters are carried out by Scanlan and Jones (1999) and Scanlan (2000, 2001). The concept is that vertical gusts and vertical velocity of the bridge are similarly filtered by the structure. Therefore, the corresponding flutter derivatives may contain information on aerodynamic admittance. An analogous observation can be made in the time domain, if indicial functions are used, as underlined by Scanlan and Jones (1999). Scanlan (2000) emphasizes the same concept, defining moreover a 'corrected' aerodynamic admittance that includes a coherence function to take into account that a bridge is a three-dimensional object. Other relationships including admittance functions related to the aerodynamic moment are proposed by Scanlan (2001). In each case, aerodynamic admittance is related to flutter derivatives associated with the vertical motion of the deck.

An alternative procedure to define admittance functions from flutter derivatives is proposed by Hatanaka and Tanaka (2002). All flutter derivatives, and not only the ones related to vertical motion, are used to calculate the admittance functions.

Experimental identification of lift admittance functions and comparisons with Sears' function are due to Jancauskas and Melbourne (1986), Sankaran and Jancauskas (1992) and Kawatani and Kim (1992). In particular, these research groups worked on the identification of lift aerodynamic admittance in smooth and turbulent flow for rectangular cylinders, with different experimental techniques, as free-decay tests and active gust generators. Jancauskas and Melbourne (1986) performed experiments in smooth flow, identifying a trend of lift admittance functions with a dimensional ratio. Sankaran and Jancauskas (1992) analysed admittance functions for rectangular sections invested by flows with different turbulence intensities, pointing out a correspondence between high frequency turbulence and lift admittance functions typical of streamlined sections. Kawatani and Kim (1992) used an active gust generator to calculate the response, confirming basically the main results of Sankaran and Jancauskas (1992).

A comparison between buffeting response of a FP and different streamlined bridge deck sections is carried out by Larose and Livesey (1997). Lift admittance functions of the Pont de Normandie and the Höga Kusten bridge are measured, pointing out a strong decaying under the unity value for low frequencies. Such effects are further investigated by Larose and co-workers, with the identification of a significant under-the-unity plateau for low frequencies of excitation, for a class of streamlined sections. This effect is modelled by means of a two-wavenumber aerodynamic admittance which takes into account some aspects of the three-dimensional character of the gusts (Larose and Mann, 1998). The effect due to the ratio between the size of the gusts and the characteristic dimension of the body is pointed out by Larose (2005).

Lift and moment admittance functions are defined by Diana and co-workers as complex functions, characterized by amplitude and phase shift. The measured admittances are introduced in the expression of buffeting forces and superimposed on self-excited loads. An experimental check of the applicability of the superposition principle is carried out, comparing admittance functions measured in bi-harmonic and mono-harmonic flows (Diana et al., 2002).

Further comparisons between wind tunnel measurements of self-excited coefficients and real part of corresponding admittance functions evidence a similar trend of self-excited coefficients and corresponding admittance functions at different angles of attack (Diana et al., 2004).

Apart from numerical simulations performed by Diana and co-workers for special cases with parameters measured in wind tunnel, complete simulations of bridge dynamics in the time domain are performed modelling buffeting forces adopting Sears' function for vertical turbulence. An approximation of admittance functions and flutter derivatives via rational functions, rather than via indicial functions, is proposed, for example, in Chen et al. (2000).

This load model takes into account aerodynamic coupling, but retains, for numerical simulations, the traditional formulation of aerodynamic admittance, because of the lack of experimental data for the calculation of buffeting parameters. This model, which can account for nonlinear aerodynamics, is further investigated by Chen and Kareem (2001). Wind actions are split into two parts, depending on their frequency, and low and high frequency components are identified. In particular, buffeting forces are calculated as high frequency components and linearized around the steady angle of attack.

1.3. Issues addressed in this paper

Analytical and experimental studies [e.g., Scanlan (2001), Hatanaka and Tanaka (2002)] suggest a way to treat the problem of admittance functions for bridges, in the sense that functions based on experimental data like flutter derivatives can be used in the definition of buffeting loads, and they are preferable to the commonly employed Sears' function. This suggestion is followed in the present work. Moreover, admittance functions are calculated from indicial functions rather than from flutter derivatives. In this way, the advantages of a time-domain formulation are preserved. In fact, although the connection between flutter derivatives and relevant admittances is assured by the Fourier transform, when sufficient data are available, the use of the transform itself would imply a frequency representation instead of a description in the time domain. It is shown that admittance functions obtained from indicial functions are a good approximation of experimentally obtained admittance functions, at least for rectangular sections. The estimate of admittance functions having the structure of Sears-like functions can be compared with experimental results provided by Jancauskas and Melbourne (1986). Qualitative agreement is observed for sections with different dimensional ratios. By this approach, the possibility of the use of parameters typical of self-excited forces to model turbulent action with an acceptable error is investigated and a complete model of wind loads based on indicial functions is presented. Scanlan and co-workers suggest the use of a couple of flutter derivatives to calculate lift admittance, while Hatanaka and Tanaka (2002) propose a model including a set of four flutter derivatives to calculate the same admittance functions.

A new interpretation of these results is provided, in the sense that the choice of the best model depends on sectional geometry. In fact, a point of conjunction can be established between the two different approaches and related to the geometry of the body under examination.

Finally, a simplified model including only vertical buffeting forces in the context of a time-domain framework for the simulation of aeroelastic behaviour of bridge decks is proposed. In the perspective of an extended time-domain analysis of a full bridge, with direct consideration of aerodynamic coupling, a complete cross-sectional analysis is performed and the response of a two-dimensional rectangular section is calculated.

The point of view being strictly two-dimensional, drag forces and longitudinal turbulence components are neglected. Span-wise coherence effects are accounted for.

2. Mechanical system

A rectangular symmetric cross-section represents here an elementary strip of a bridge deck (Fig. 1). The main characteristic dimension is assumed to be the width of the deck section B, referred to as the chord. The half-chord is indicated by $b = B/2$. The thickness of the section is indicated by D, the span of the bridge by l.

The dimensional ratio B/D between chord and thickness is a parameter that describes the slenderness of the structure, i.e. the optimization in the aerodynamic sense. Even if a rectangular section shows separation of flow as result of sharp edges, for a section sufficiently elongated, flow reattaches and the aerodynamic behaviour should be similar to that of an ideal FP. An 'optimized' section is referred to also as streamlined, in contrast to a bluff section, in which separation of shear layers strongly affects the vortex street. Quasi-steady theory can qualitatively describe dynamic behaviour of streamlined sections, while unsteady effects become fundamental for bluff sections [see, for example, the quasi-steady analyses carried out by Costa and Borri(2006) on rectangular sections].

The body depicted in Fig. 1 is supposed to have only a vertical degree of freedom y (*heaving*) and a rotational degree of freedom α (*torsion*). Horizontal displacements are neglected. The body is considered to be a rigid body: mass is assumed to be concentrated at the centre of mass G and the elastic properties are represented by translational springs with stiffness k, coupled with dampers with damping constant c, connected to the elastic centre E at a certain distance d. As a result of the assumed symmetry of the section, the elastic centre E and the centre of mass G are coincident. They are both located at the midspan of the deck section. The spring-damper pairs give to the section a vertical stiffness $k_y = 2k$, a torsional stiffness $k_\alpha = 2d^2k$, a vertical damping $c_y = 2c$ and a torsional damping $c_\alpha = 2d^2c$.

Two reference frames are defined: first a Cartesian inertial reference frame $OXYZ$, and second a coordinate system $ox'y'z'$, attached to the vibrating structure, with the origin o coincident with the centre of mass G of the section and the x' and the y' axes oriented, respectively, along the sectional middle-line chord and the orthogonal direction.

The reference frames are assigned by following the 'airfoil convention', namely positive vertical translations are directed downwards and positive rotations are clockwise (nose-up).

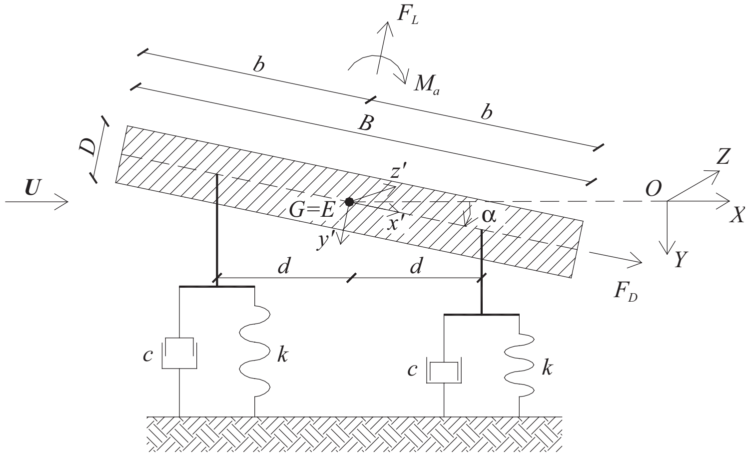

Fig. 1. Cross-section schematic (B = chord; b = half-chord; D = thickness; k = stiffness; c = damping constant), relevant points (G = centre of mass; E = elastic centre), wind forces (F_L = lift force; F_D = drag force; M_a = aerodynamic moment), wind flow (U = wind velocity), global reference system ($OXYZ$) and local reference system ($o'x'y'z'$).

The equations of motion describing the dynamic system are given by

$$m\ddot{y} + c_y\dot{y} + k_y y = F_Y, \tag{1}$$

$$I\ddot{\alpha} + c_\alpha\dot{\alpha} + k_\alpha\alpha = M_Z, \tag{2}$$

where m is the mass and I is the mass momentum of inertia per unit length.

The wind load is modelled as an external action, namely a live load. In particular, the resulting force acting on the section is decomposed as the combination of a lift force F_L, a drag force F_D and an aerodynamic moment M_a, applied on the centre of mass G. Lift and drag forces act, respectively, along the y' and the x' axes. The aerodynamic moment acts about the $z' \equiv Z$-axis directed along the bridge span, namely $M_a \equiv M_Z$. Vertical force F_Y is obtained by combining lift and drag forces with reference to the global coordinate system:

$$F_Y = -F_L \cos\alpha + F_D \sin\alpha. \tag{3}$$

The 'airfoil convention' is adopted also in the wind load representation, that is the lift force F_L is positive upwards and the aerodynamic moment M_a is positive clockwise (nose-up).

Linearized theory is used: the angle of attack α, defined as the angle between the section chord and the main flow direction, is small. This allows one to assume that $|F_L| = |F_Y|$. The lift force and aerodynamic moment are considered in a global reference frame in what follows. Accordingly, the components of motion are expressed in global coordinates. Such a point of view is often assumed implicitly in linear aerodynamics, when one considers only one reference system that is the global one.

3. Wind field

The section is immersed in a turbulent wind field. Wind is assumed incompressible and nonviscous. The wind velocity U_w can be decomposed additively as a time-space variable field

$$U_w(M,t) = U(M) + \hat{U}(M,t), \tag{4}$$

where t is the time, U is the mean wind speed and $\hat{U}$ represents a turbulent perturbation, acting at point M of coordinates X_M, Y_M and Z_M.

The mean wind is directed along the X-axis. In this simplified case, only the vertical component of turbulence w, directed along the vertical axis Y, is accounted for. It is assumed that such a turbulence component is mainly responsible for buffeting forces in a planar scheme. The resultant wind field is then given by

$$U_{wX}(M,t) = U(M),\ U_{wY}(M,t) = \mathsf{w}(M,t),\ U_{wZ}(M,t) = 0. \tag{5}$$

Assuming neutral conditions, the mean wind velocity U may be expressed by the logarithmic profile, while turbulence components are defined through an assigned power-spectral density function. Lumley and Panowsky's normalized spectrum is assumed for vertical turbulence (Simiu and Scanlan, 1996)

$$\frac{fS_{\mathsf{w}}(f)}{u_*^2} = \frac{3.36(L_{ux}f/U)}{1 + 10(L_{ux}f/U)^{5/3}}, \tag{6}$$

where L_{ux} is the integral length scale, f is the frequency and u_*^2 is the friction velocity, which depends on roughness and takes into account shear stresses.

4. Time-dependent sectional forces

If the superposition principle is assumed to apply and drag components are neglected, the wind load acting on a section can be expressed as

$$F_L(t) = F_{L\mathrm{b}}(t) + F_{L\mathrm{se}}(t), \tag{7}$$

$$M_a(t) = M_{a\mathrm{b}}(t) + M_{a\mathrm{se}}(t), \tag{8}$$

where subscripts b and se indicate, respectively, buffeting and self-excited forces.

In this case, dead loads and static wind forces are neglected.

4.1. Buffeting forces

For bridges, lift and moment buffeting forces $F_{L\mathrm{b}}$ and $M_{a\mathrm{b}}$ can be written, in the mixed time–frequency domain and considering only vertical harmonic disturbances w (Scanlan and Jones, 1990), as

$$F_{L\mathrm{b}}(t) = \frac{1}{2}\rho U^2 B \frac{\mathrm{d}C_L}{\mathrm{d}\alpha}\chi_{L\mathsf{w}}(k)\frac{\mathsf{w}}{U}, \tag{9}$$

$$M_{a\mathrm{b}}(t) = \frac{1}{2}\rho U^2 B^2 \frac{\mathrm{d}C_M}{\mathrm{d}\alpha}\chi_{M\mathsf{w}}(k)\frac{\mathsf{w}}{U}, \tag{10}$$

where ρ is the air density, $k = \omega b/U$ is the reduced frequency, $C_L(\alpha)$ and $C_M(\alpha)$ are the aerodynamic coefficients defined as functions of the angle of attack. The admittance functions $\chi_{L\mathsf{w}}(k)$ and $\chi_{M\mathsf{w}}(k)$ represent the transfer functions between the turbulent component w and the sectional forces. In the quasi-steady formulation, the admittance functions take the value of unity.

For a thin airfoil encountering a sinusoidal vertical gust $\mathsf{w}(t) = \mathsf{w}_0 \exp(\mathrm{i}\omega t)$, lift buffeting force is expressed via Sears' function

$$\chi(k) = C(k)[\mathrm{J}_0(k) - \mathrm{i}\mathrm{J}_1(k)] + \mathrm{i}\mathrm{J}_1(k), \tag{11}$$

obtained by combining Bessel functions of first and third kind $\mathrm{J}_0(k)$ and $\mathrm{J}_1(k)$ and Theodorsen's complex circulation function $C(k) = F(k) + \mathrm{i}G(k)$.

Sears' function $\chi(k)$ is the lift admittance function for thin airfoils, i.e. the frequency-based transfer function which relates vertical velocity fluctuations due to turbulent wind to the lift force and moment fluctuations experienced by the structure. Gust action is assumed as fully correlated along the span. The buffeting lift is calculated as $F_{L\mathrm{b}}(t) = 2\pi\rho Ub\chi(k)\mathsf{w}(t)$.

In order to account for arbitrary gusts, superposition of sharp-edged gusts may be considered. Dimensionless lift developing on an airfoil due to a sharp-edged gust of amplitude w_0 striking the leading edge of the airfoil at $s = 0$ can be described through Küssner's function $\psi(s)$, a function representing the counterpart of Sears' one in the time domain, as $F_{L\mathrm{b}}(s) = 2\pi\rho Ub\mathsf{w}_0\psi(s)$, where s is dimensionless time defined by $s = Ut/b$ (Fung, 1969).

An approximation of Küssner's function in incompressible flow, for $s \geqslant 0$, is given by $\psi(s) \simeq 1 - 0.500\mathrm{e}^{-0.130s} - 0.500\mathrm{e}^{-s}$.

4.2. Self-excited forces

The common expression for self-excited forces is in a mixed time–frequency domain, considering a sinusoidal coupled motion of reduced frequency $K = \omega B/U$ (Scanlan and Jones, 1990):

$$F_{L\mathrm{se}}(t) = \frac{1}{2}\rho U^2 B\left[KH_1^*(K)\frac{\dot{y}(t)}{U} + KH_2^*(K)\frac{B\dot{\alpha}(t)}{U} + K^2H_3^*(K)\alpha(t) + K^2H_4^*(K)\frac{y(t)}{B}\right], \tag{12}$$

$$M_{a\mathrm{se}}(t) = \frac{1}{2}\rho U^2 B^2\left[KA_1^*(K)\frac{\dot{y}(t)}{U} + KA_2^*(K)\frac{B\dot{\alpha}(t)}{U} + K^2A_3^*(K)\alpha(t) + K^2A_4^*(K)\frac{y(t)}{B}\right], \tag{13}$$

where ρ is the air density and H_i^* and A_i^* $(i = 1,\ldots,4)$ are the flutter derivatives, commonly identified in wind tunnel tests as function of reduced frequency K or reduced velocity $U_{\mathrm{red}} = 2\pi/K = 2\pi U/\omega B$. These expressions of self-excited loads follow '*Scanlan's convention*', namely lift force is positive downwards and aerodynamic moment is positive clockwise (nose-up).

Self-excited forces can be defined in a pure time-domain framework by means of *indicial functions*, alternatively as a function of t or s, as common in wing aerodynamics. By following the expressions analysed by Costa and Borri(2006) for a set of rectangular sections, the lift force and moment can be defined, respectively, as follows:

$$F_{L\mathrm{se}}(s) = \frac{1}{2}\rho U^2 B\frac{\mathrm{d}C_L}{\mathrm{d}\alpha}\left[\Phi_{L_y}(0)\frac{2}{B}y'(s) + \Phi_{L_\alpha}(0)\alpha(s) + \int_0^s \Phi'_{L_y}(s-\sigma)\frac{2}{B}y'(\sigma)\,\mathrm{d}\sigma + \int_0^s \Phi'_{L_\alpha}(s-\sigma)\alpha(\sigma)\,\mathrm{d}\sigma\right], \tag{14}$$

$$M_{a\mathrm{se}}(s) = \frac{1}{2}\rho U B^2\frac{\mathrm{d}C_M}{\mathrm{d}\alpha}\left[\Phi_{M_y}(0)\frac{2}{B}y'(s) + \Phi_{M_\alpha}(0)\alpha(s) + \int_0^s \Phi'_{M_y}(s-\sigma)\frac{2}{B}y'(\sigma)\,\mathrm{d}\sigma + \int_0^s \Phi'_{M_\alpha}(s-\sigma)\alpha(\sigma)\,\mathrm{d}\sigma\right], \tag{15}$$

where Φ_{hk} are the indicial functions ($h = L$, M; $k = y$, α). Primes denote differentiation with respect to s.

Each indicial function describes the evolution of the resulting action as a consequence of a sudden change in angle of attack α or vertical velocity y'. It is defined as the sum of a constant part a_{0hk} and n exponential groups characterized by the pairs of coefficients (a_{ihk}, b_{ihk}):

$$\Phi_{hk}(s) = a_{0hk} - \sum_{i=1}^{n} a_{ihk} \exp(-b_{ihk}s), \tag{16}$$

or, alternatively, in the time domain,

$$\hat{\Phi}_{hk}(t) = a_{0hk} - \sum_{i=1}^{n} a_{ihk} \exp\left(-b_{ihk} \frac{2U}{B} t\right). \tag{17}$$

For a thin airfoil in sinusoidal motion, self-excited lift force due to circulation can be calculated by means of Theodorsen's circulation function as $F_{L\mathrm{se}}(t) = 2\pi\rho UbC(k)w(t)$, where $w(t) = \dot{y} + U\alpha + \frac{1}{2}b\dot{\alpha}$ is the *downwash*, that is the vertical velocity of the wind particle in contact with the special point at the three-quarter chord distance from the leading edge (rear point).

The counterpart in the time domain of Theodorsen's function corresponds to Wagner's indicial function $\phi(s)$, which depicts the growth of the lift force on an airfoil due to a sudden unit change on the angle of attack α_0. Self-exited lift is expressed by $F_{L\mathrm{se}}(s) = 2\pi\rho Ub\alpha_0\phi(s)$.

The following exponential approximation of Wagner's function due to Jones is usually adopted $\phi(s) \simeq 1 - 0.165\mathrm{e}^{-0.0455s} - 0.335\mathrm{e}^{-0.30s}$.

A similar correspondence can be established for bridges between indicial functions Φ_{hk} and equivalent complex circulatory functions $C_{hk\mathrm{eq}}(k) = F_{hk\mathrm{eq}}(k) + \mathrm{i}G_{hk\mathrm{eq}}(k)$.

In this load model, a group of four indicial functions Φ_{hk} is considered, therefore four equivalent functions $C_{hk\mathrm{eq}}(k)$ are introduced. The group of four indicial functions allows maximum flexibility in the representation of self-excited forces.

The model proposed by Hatanaka and Tanaka (2002) accounts, on the contrary, only for two equivalent Theodorsen's functions ($C_{L\mathrm{eq}}(k)$ and $C_{M\mathrm{eq}}(k)$), referring, respectively, to lift force and aerodynamic moment.

In the special case of thin airfoil, only one Wagner's function is adequate to define both self-excited lift and moment in time domain: the components of downwash w (i.e. vertical velocity, torsional displacement and torsional velocity) are accounted for equally.

Therefore, as the geometry of the section approaches the airfoil, the number of parameters needed to describe unsteady forces is reduced accordingly, by considering both indicial functions and exponential groups. Two indicial functions with one exponential group or one indicial function with two exponential groups are equivalent from the point of view of the number of parameters necessary to capture aerodynamic features and they correspond to the limit case of FP.

It is common that complex geometries like bridge deck sections require more than one indicial function to represent unsteady effects. Moreover, different functions underline the different role played by each component of downwash.

In this sense, a model accounting for one function interpolating, at the same time, all four flutter derivatives for lift and moment is more suitable in the case of very streamlined sections, while more than one function becomes necessary for bluff bodies.

The identification of indicial function coefficients is performed via a nonlinear least-square method. A Nelder-Mead simplex algorithm is adopted. Once experimental aeroelastic derivatives are assigned, a prescribed number of exponential groups characterizing each indicial function is fixed, and appropriate coefficients are identified by minimizing a norm, defined, for example, as the two-norm of the difference between estimated and experimental values. Relevant details are given in Costa and Borri(2006). Results of the identification procedure are sensitive to the starting data, with dependence, in particular, on the range of reduced velocities covered by the data and on dispersion of measurements. A weighting function can be included to account for significance of different flutter derivatives.

5. Use of indicial functions to define buffeting loads

An approximate approach to define buffeting loads in time domain is proposed here.

First of all, it is noted that (a) a sinusoidally moving section immersed in a uniform flow and (b) a fixed section under the action of a sinusoidal gust can be treated in a similar way, at least within the limits of a linearized approach [see Tubino (2005)]. Even if situations (a) and (b) give rise to different physical phenomena and local distributions of pressures, it is assumed as a *working hypothesis* that, under certain conditions, the integral measure of forces could exhibit no great differences, and that the measured quantities called flutter derivatives may contain some information also on the estimate of the admittance, at least in a pure cross-sectional analysis with a fully coherent gust.

Moreover, by the observation of similarity of Wagner's and Küssner's functions, another suggestion arises, that is, in the time domain, for bridge deck cross-sections, indicial functions representative of self-excited and buffeting forces could be not too far from each other, and as a limit condition they could be "*look-alikes*". Then, the investigation of what happens if indicial functions obtained for self-excited forces are also used to define buffeting loads appears to be an interesting subject.

For a thin airfoil, the use of indicial functions obtained for self-excited forces leads to incorrect results, because self-excited and buffeting forces are fully reproduced through the well-defined Wagner's and Küssner's functions. But, for the purposes of aeroelastic analysis of bridges, with special attention to cross-sectional models tested in wind tunnel, an approximate formulation of the theory could be accepted, in which self-excited and buffeting loads are reproduced by means of equal indicial functions. In any case, in the absence of measured admittances, Sears' function is often adopted. A tool to define a function accounting, in some way, for the specific geometrical features of the section is proposed here.

The action due to turbulent wind components is expressed by

$$F_{L\mathrm{b}}(s) = \frac{1}{2}\rho U^2 B C'_L \left[\Phi_{Ly}(0)\frac{2}{B}\mathrm{w}(s) + \int_0^s \Phi'_{Ly}(s-\sigma)\frac{2}{B}\mathrm{w}(\sigma)\,\mathrm{d}\sigma \right], \tag{18}$$

$$M_{a\mathrm{b}}(s) = \frac{1}{2}\rho U^2 B^2 C'_M \left[\Phi_{My}(0)\frac{2}{B}\mathrm{w}(s) + \int_0^s \Phi'_{My}(s-\sigma)\frac{2}{B}\mathrm{w}(\sigma)\,\mathrm{d}\sigma \right]. \tag{19}$$

Indicial functions have, in this context, the meaning of dimensionless forces developing on the section due to a vertical gust.

In order to validate this approximate approach and/or to search its limits, two steps are performed: (i) the identification of indicial coefficients, (ii) the calculation of an equivalent Theodorsen's function and equivalent admittance.

As a first step, indicial functions are obtained through a nonlinear least-squares procedure from flutter derivatives. Coefficients of equivalent Theodorsen's functions are calculated by the following explicit relationships:

$$F_{hk\mathrm{eq}}(k) = 1 - \sum_i^n \frac{a_{ihk}k^2}{b_{ihk}^2 + k^2}, \tag{20}$$

$$G_{hk\mathrm{eq}}(k) = -\sum_i^n \frac{a_{ihk}b_{ihk}k}{b_{ihk}^2 + k^2}. \tag{21}$$

Then, admittance functions for bridges $\chi_{\mathrm{eq}}(k)$ are calculated as equivalent Sears-like functions, then as combinations of equivalent Theodorsen's functions $C_{hk\mathrm{eq}}(k)$ and Bessel functions. In particular, two admittance functions are calculated, one for the lift and one for the aerodynamic moment:

$$\chi_L(k) = C_{Ly\mathrm{eq}}(k)[\mathrm{J}_0(k) - \mathrm{i}\mathrm{J}_1(k)] + \mathrm{i}\mathrm{J}_1(k), \tag{22}$$

$$\chi_M(k) = C_{My\mathrm{eq}}(k)[\mathrm{J}_0(k) - \mathrm{i}\mathrm{J}_1(k)] + \mathrm{i}\mathrm{J}_1(k). \tag{23}$$

The validation of the model is based on the comparison of estimated aerodynamic admittances with experimental results, in cases in which they are available.

The procedure is applied to different rectangular sections, identified by dimensional ratios B/D. Flutter derivatives for all sections are taken from Matsumoto et al. (1996). Interesting issues on torsional flutter mechanism of 2-D rectangular cylinders can be found in Matsumoto et al. (1997).

Some remarks on the proposed model: Sankaran and Jancauskas (1992) point out that the level of turbulence intensity affects admittance functions. In this case, admittance functions do not take into account explicitly turbulence intensity, but, on the other hand, they are obtained from indicial functions, which are different if obtained under different turbulence conditions. Therefore, in principle, the effect of turbulence intensity is already included in self-excited parameters; the approach presented can be assumed valid only for fully correlated sinusoidal gusts, which provide only a simplified representation of natural turbulence. On the other hand, it is important to underline that aerodynamic admittances can be successfully identified with a numerical procedure starting from indicial functions, and that the suggested tool can be used confidently to identify admittance functions preferable to Sears' function.

5.1. Admittance functions

Lift and moment admittance functions are calculated for a set of rectangular sections characterized by different dimensional ratios: $B/D = 5$; 8; 10; 12.5; and 15. First of all, indicial functions are evaluated and corresponding equivalent Theodorsen's functions are calculated according to Eqs. (20) and (21). Approximation of lift flutter derivatives is performed successfully with indicial functions characterized by one exponential group. Moment flutter derivatives require an approximation with two exponential groups for the bluffest sections, in this case $B/D = 5$, 8 and 10. This fact influences strongly the results, especially for moment admittance functions.

5.1.1. Lift admittance functions

Lift admittance functions are calculated according to Eq. (22). Results are shown in Fig. 2, together with the reference Sears' function, i.e. the lift admittance corresponding to thin airfoil. As expected, the increasing of

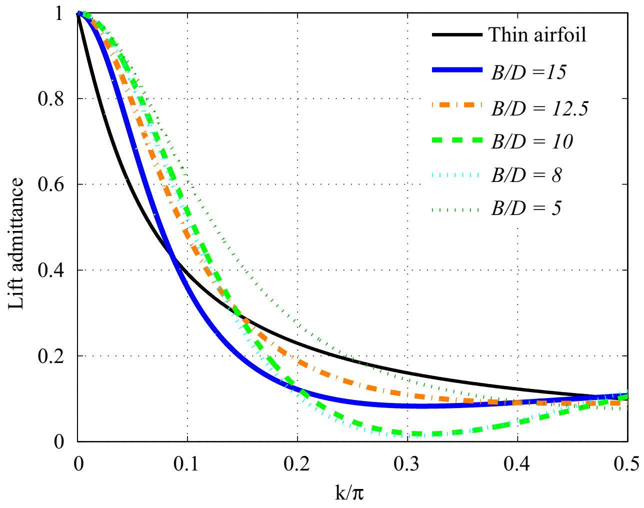

Fig. 2. Lift admittance functions for rectangular sections.

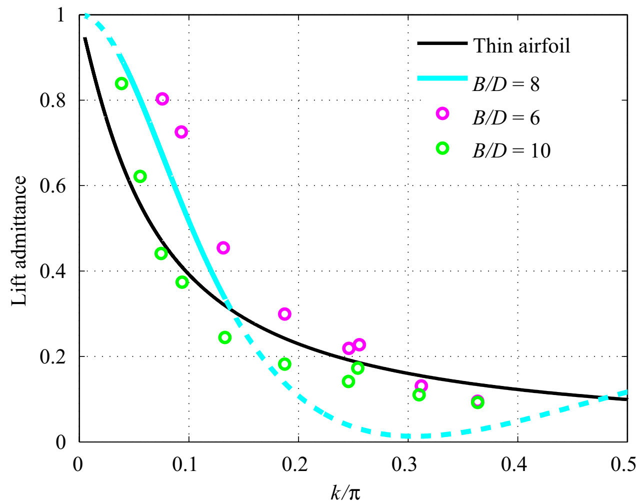

Fig. 3. Comparison of admittance functions calculated by means of indicial functions for a rectangular section with $B/D = 8$ with Sears' function and experimental results (Jancauskas and Melbourne, 1986).

dimensional ratio B/D corresponds to a qualitative similarity with Sears' function, especially in the low frequency range.

In particular, results by the identification procedure have been compared with experimental results taken from Sankaran and Jancauskas (1992) and Jancauskas and Melbourne (1986). They derive lift admittance functions with two different experimental techniques and both results are in excellent agreement with each other. The conditions of the experimental set-up are, in both cases, those required by the application of the proposed method, i.e. admittances are obtained under fully correlated gusts. Results are plotted in Figs. 3, 6 and 7.

It is worth recalling that the applied procedure is supposed to give an *estimate* of admittance functions, and that experimental data used for calculation and validation of the procedure are obtained in a different wind tunnel, under different conditions.

In all cases, the region of reduced velocities $U_{\mathrm{red}} = [7.410, 22.24]$ (i.e. of reduced frequencies $k/\pi = [0.045, 0.135]$) corresponds to the range for which flutter derivatives are available. Corresponding admittances are indicated by solid lines. In the present work, flutter derivatives in the range of reduced velocity $U_{\mathrm{red}} = [0, 7.410]$ (i.e. of reduced frequencies k/π over 0.135) are only extrapolated. Corresponding admittances are indicated by dashed lines. For very

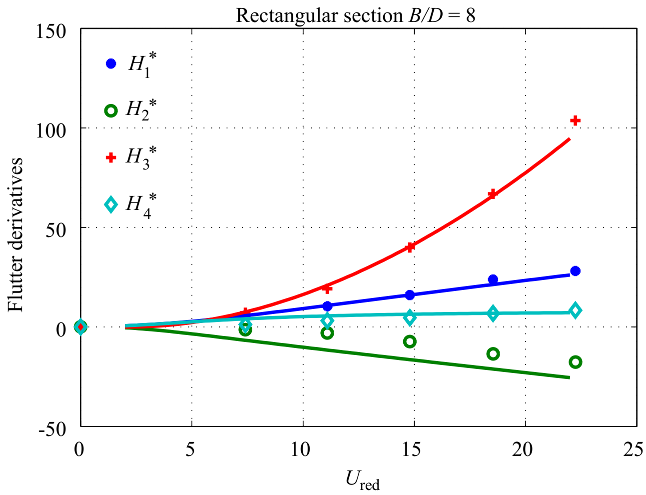

Fig. 4. Experimental flutter derivatives (discrete values) and their approximation for $B/D = 8$ obtained by means of indicial functions (solid lines).

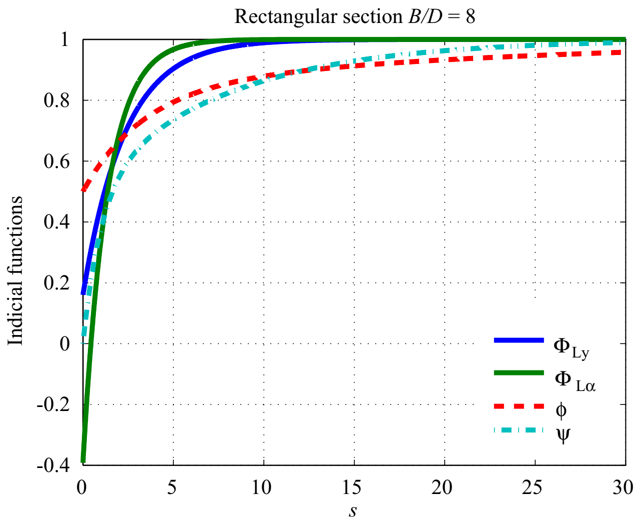

Fig. 5. Indicial functions for $B/D = 8$ compared with Sears' and Küssner's functions.

high reduced velocities, in this case for $U_{red} > 22.24$ (i.e. $k/\pi < 0.045$), the admittance functions are also indicated by dashed lines.

For the $B/D = 8$ section, lift admittance calculated via indicial functions (thick line) is compared with Sears' functions (thin line) and experimental values (circles) of rectangular sections $B/D = 10$ and 6 (Fig. 3). In the range corresponding to the availability of measured coefficients, a good interpolation of the measured admittances is achieved. In the low reduced velocity range, the identified admittance does not interpolate the experimental results. In fact, also for flutter derivatives, values are extrapolated in such an interval, and the admittance is expected to be extrapolated too. All the measured data remain above the identified curve, which represents only a ''*convex envelope*'' of the measured results.

As an example, flutter derivatives corresponding to $B/D = 8$ are plotted in Fig. 4. Five values are known for each flutter derivative, in the range $U_{red} = [7.410, 22.24]$. For the identification procedure, a number of fictitious

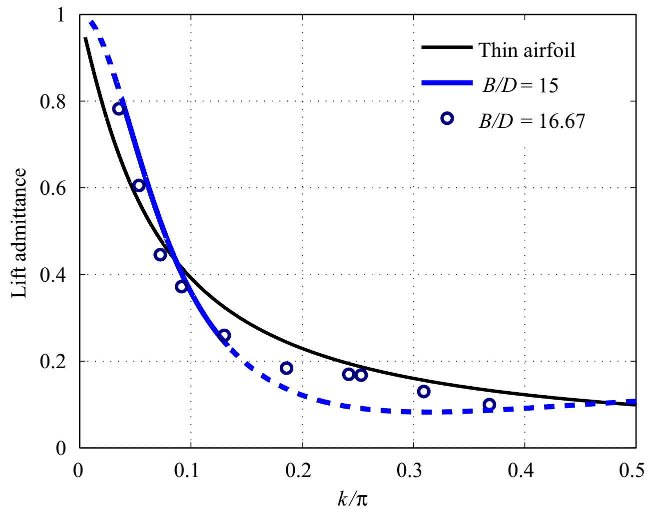

Fig. 6. Comparison of admittance function calculated by means of indicial functions for a rectangular section with $B/D = 15$ with Sears' function and experimental results (Jancauskas and Melbourne, 1986).

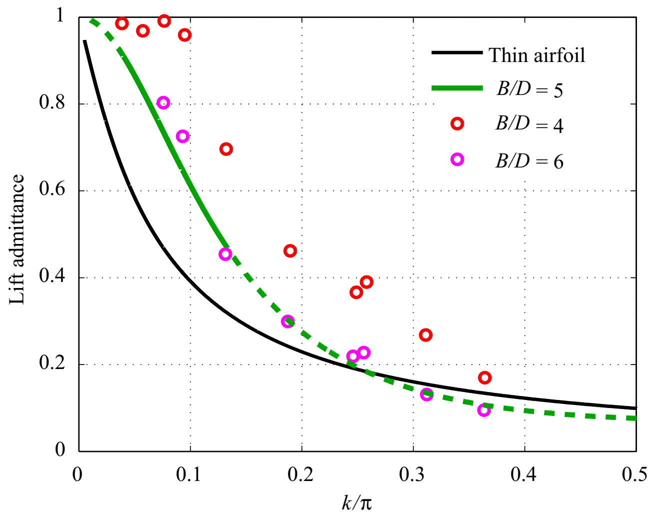

Fig. 7. Comparison of admittance function calculated by means of indicial functions for a rectangular section with $B/D = 5$ with Sears' function and experimental results (Jancauskas and Melbourne, 1986).

experimental data is generated, once interpolating polynomial functions are assigned. Solid lines appearing in Fig. 4 represent the approximation of flutter derivatives obtained by means of indicial functions, which are shown in Fig. 5, in comparison with Wagner's and Küssner's functions.

The lift admittance corresponding to the most streamlined rectangular section $B/D = 15$ is compared with $B/D = 16.67$ (Fig. 6), while the bluffest section treated (i.e. the rectangular section with $B/D = 5$) is compared with experimental results valid for $B/D = 4$ and 6 (Fig. 7). According to previous observation, dashed parts of the curves refer to extrapolated data. Trend followed by the identified admittances is consistent with the experiments. Based on all the examples considered, it can be observed that, for the low reduced velocities under 7.410, the admittance curve obtained from indicial functions represents always a ''*convex envelope*'' for the possible experimental data. Based on these results, a conservative estimation of the admittance can be practically obtained by combining admittances estimated by means of indicial functions in the low-frequency range and Sears' function in high frequency range.

Such results could be improved if flutter derivatives and, as a consequence, indicial functions, were available in a wider range.

Qualitative comparison can be performed also with results provided by Scanlan (2000,2001) for bridge deck cross-sections.

5.1.2. Moment admittance functions

Moment admittance functions are calculated by following the same procedure, according to Eq. (23). Results are shown in Fig. 8. In both cases, for lift and moment admittance functions, all curves shrink to zero, by letting k go to infinity.

The identification of the moment admittance function may be a matter of discussion, due to the lack of experimental data. Some comparisons can be performed with literature results, provided, for example, by Scanlan (2000, 2001).

In particular, moment admittances greater than unity and with shapes very different from Sears' function have been obtained by Scanlan (2000).

Interesting results confirming the previous ones are also given by de la Foye (2001), who measured, for a rectangular section with dimensional ratio $B/D = 8$, a moment aerodynamic admittance three times greater than unity.

Nevertheless, an important remark should be added on the moment admittances of sections $B/D = 5$, 8 and 10, that show an unusual behaviour. They have in common the fact that indicial functions necessary to capture the behaviour of the flutter derivatives require two exponential groups (which is surely a numerical matter, but it is strongly suggested by the distribution of the experimental data; in this sense, the choice of exponential groups contain indirectly some physical information). This circumstance suggests that unsteady effects involved in self-excited forces are significant and cannot be disregarded. It could be possible that such unsteady effects influence also lift derivatives used to calculate the admittance. In this case, the physical phenomena involved could be too far from the applicability conditions of the method and compromise the results. In this sense, the number of groups necessary to identify the indicial functions could define a limit for the applicability of the proposed method.

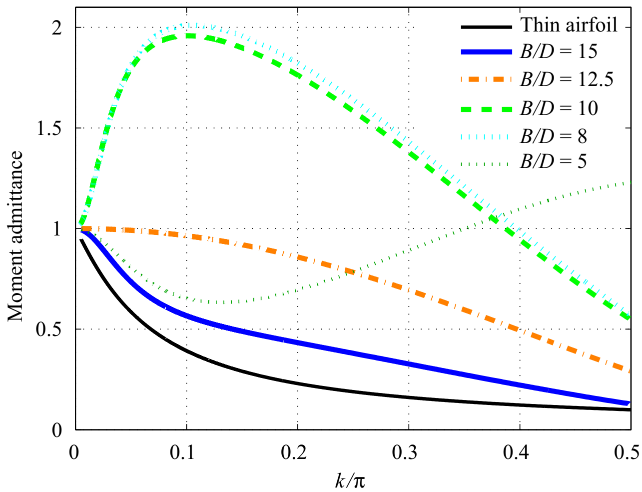

Fig. 8. Moment admittance functions for rectangular sections.

6. Numerical simulations

The complete load model represented by Eqs. (18) and (19) is applied to a sample rectangular section with dimensional ratio $B/D = 12.5$, together with self-excited forces described by (14) and (15). In this model, buffeting and self-excited forces are treated with an analogous formulation which accounts for process unsteadiness. The aim of this section is to clarify the differences, in terms of dynamic behaviour, of the indicial model with respect to the quasi-steady one.

Once the Lumley and Panofsky spectrum is assigned to vertical turbulence (Simiu and Scanlan, 1996), the following analyses are performed: (i) quasi-steady buffeting; (ii) unsteady buffeting with equal indicial functions for self-excited and buffeting loads.

Mechanical properties of the analysed cross-section are recalled in Table 1.

Indicial functions and aerodynamic coefficients obtained from flutter derivatives are recalled in Tables 2 and 3. Only one exponential group is sufficient in order to well capture the features of the eight flutter derivatives.

Numerical simulations are performed over a time interval of 8.192 s, with a time step of 5×10^{-4} s.

Root-mean-squares and maximum values for heaving and torsional displacements are obtained in both cases, at different wind speeds (Figs. 9 and 10, respectively). They can be compared with results provided by Bartoli and Righi (2006).

A response overestimation is expected in considering both load models, because of the effective loss of coherence in incoming wind due to three-dimensional effects, as underlined, for example, by Matsuda et al. (1999).

In particular, by considering heaving displacements, the quasi-steady buffeting model overestimates the sectional response, while the model based on indicial functions seems to capture the effective sectional behaviour. If torsion displacements are considered, a major overestimation of the response is observed in indicial model results. This fact is mainly due to self-excited forces and to the restricted number of experimental data available in the range of reduced velocities of interest. This effect is already observed in smooth flow, but results amplified in the case of turbulent flow. Nevertheless, it can be easily removed if a more detailed extraction of aeroelastic data is available. Perfect frequency

Table 1
Geometrical and mechanical properties

l (m)	b (m)	ω_y (rad s^{-1})	f_y (Hz)	m (kg m^{-1})	ζ_y
0.920	0.1875	36.88	5.87	3.810	0.0018
D (m)	B (m)	ω_α (rad s^{-1})	f_α (Hz)	I (kg m^2 m^{-1})	ζ_α
0.03	0.375	52.15	8.30	0.037	0.0028

Rectangular section $B/D = 12.5$.

Table 2
Indicial functions

IF	a_1	b_1
Φ_{Ly}	0.9711	2.146
$\Phi_{L\alpha}$	1.0218	0.6636
Φ_{My}	0.2022	19.6084
$\Phi_{M\alpha}$	0.9541	2.0731

Rectangular section $B/D = 12.5$.

Table 3
Dynamic derivatives

C'_L	6.48
C'_M	1.04

Rectangular section $B/D = 12.5$.

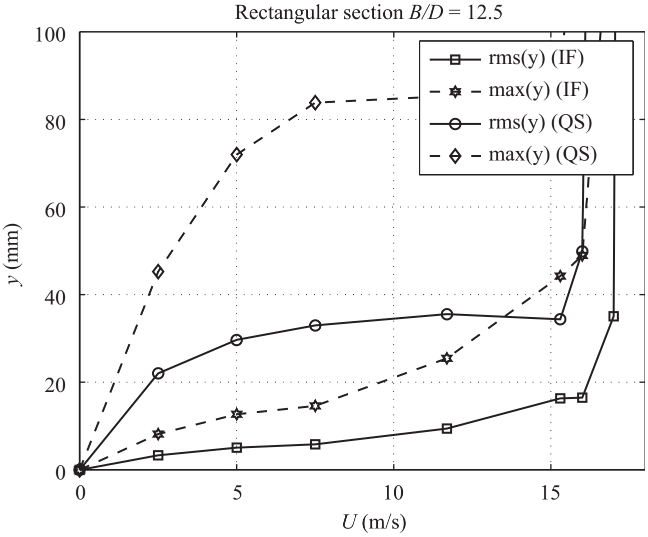

Fig. 9. Quasi-steady theory versus indicial theory: heaving displacements.

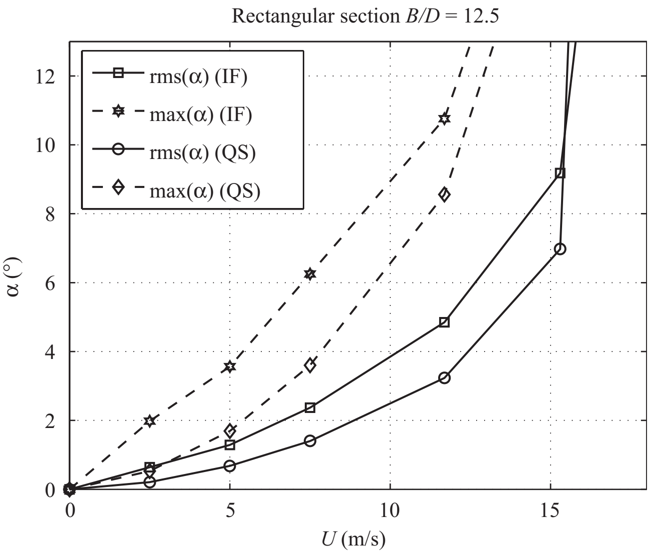

Fig. 10. Quasi-steady theory versus indicial theory: torsional displacements.

coupling is observed, in turbulent case, at a slight higher incoming wind velocity, with respect to the smooth flow case. In terms of displacements, the flutter condition becomes less evident, being not possible to identify an incoming wind velocity for which vertical and torsional displacements remain at the same amplitude. As reference values, the main results obtained in smooth flow are collected in Table 4.

7. Conclusions

In this paper, a simplified method for describing buffeting loads via indicial functions is presented. Indicial functions calculated from aeroelastic derivatives are used to filter the vertical gusts as well as the vertical velocity of the section. Admittance functions for lift and moment are calculated starting from indicial functions. The procedure is verified by comparison of equivalent admittances with experimental results. A good trend is obtained for both 'aerodynamic' and 'bluff' sections. Nevertheless, a limitation of the proposed approach is found in the identification of the moment

Table 4
Results on flutter analyses

Flutter analysis method	U_{crit} (m s^{-1})	f_{crit} (Hz)	$U_{red,crit}$
Indicial function model	15.35	7.32	5.43
Wind tunnel tests	15.54	7.37	5.75

Rectangular section $B/D = 12.5$.

aerodynamic admittances for too-bluff sections. Numerical simulations can be performed easily, including in the same framework self-excited and buffeting loads.

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References

Bartoli, G., Righi, M., 2006. Flutter mechanism for rectangular prisms in smooth and turbulent flow. Journal of Wind Engineering and Industrial Aerodynamics 94, 275–291.

Chen, X., Kareem, A., 2001. Nonlinear response analysis of long-span bridges under turbulent winds. Journal of Wind Engineering and Industrial Aerodynamics 89, 1335–1350.

Chen, X., Matsumoto, M., Kareem, A., 2000. Time domain flutter and buffeting response. Analysis of bridges. ASCE Journal of Engineering Mechanics 126 (1), 7–16.

Costa, C., Borri, C., 2006. Application of indicial functions in bridge deck aeroelasticity. Journal of Wind Engineering and Industrial Aerodynamics 94, 859–881.

Davenport, A., 1962. New estimation method of aerodynamic admittance function. Journal of Wind Engineering and Industrial Aerodynamics 90, 2073–2086.

de la Foye, A., 2001. Calcul de la réponse dynamique des structures élancées à la turbulence du vent. Ph.D. Dissertation, Université de Nantes, France.

Diana, G., Bruni, S., Cigada, A., Zappa, E., 2002. Complex aerodynamic admittance function role in buffeting response of a bridge deck. Journal of Wind Engineering and Industrial Aerodynamics 90, 2057–2072.

Diana, G., Resta, F., Zasso, A., Belloli, M., Rocchi, D., 2004. Forced motion and free motion aeroelastic tests on a newconcept dynamometric section model of the messina suspension bridge. Journal of Wind Engineering and Industrial Aerodynamics 92, 441–462.

Fung, Y.C., 1969. An Introduction to the Theory of Aeroelasticity. Dover Publications, Mineola, New York.

Hatanaka, A., Tanaka, H., 2002. New estimation method of aerodynamic admittance function. Journal of Wind Engineering and Industrial Aerodynamics 90, 2073–2086.

Jancauskas, E.D., Melbourne, W.H., 1986. The aerodynamic admittance of two-dimensional rectangular section cylinders in smooth flow. Journal of Wind Engineering and Industrial Aerodynamics 23, 395–408.

Kawatani, M., Kim, H., 1992. Evaluation of aerodynamic admittance for buffeting analysis. Journal of Wind Engineering and Industrial Aerodynamics 41–44, 613–624.

Küssner, H.G., 1936. Zusammenfassender bericht ueber den instantionaren auftrieb von fluegeln. Luftfahrtforschung 13, 410–424.

Larose, G.L., 2005. Aerodynamic admittance of bridge decks: a proposal. Proceedings of the Sixth European Conference on Structural Dynamics EURODYN2005, pp. 391–395.

Larose, G.L., Livesey, F.M., 1997. Performance of streamlined bridge decks in relation to the aerodynamics of a flat plate. Journal of Wind Engineering and Industrial Aerodynamics 69–71, 851–860.

Larose, G.L., Mann, J., 1998. Gust loading on streamlined bridge decks. Journal of Fluids and Structures 12, 511–536.

Liepmann, H.W., 1955. Extension of the statistical approach to buffeting and gust response of wings of finite span. Journal of the Aeronautical Sciences 22, 197–200.

Matsuda, K., Hikami, Y., Fujiwara, T., Moriyama, A., 1999. Aerodynamic admittance and the 'strip theory' for horizontal buffeting forces on a bridge deck. Journal of Wind Engineering and Industrial Aerodynamics 83, 337–346.

Matsumoto, M., Kobayashi, Y., Shirato, H., 1996. The influence of aerodynamic derivatives on flutter. Journal of Wind Engineering and Industrial Aerodynamics 60, 227–239.

Matsumoto, M., Daito, Y., Yoshizumi, F., Ichikawa, Y., Yabutani, T., 1997. Torsional flutter of bluff bodies. Journal of Wind Engineering and Industrial Aerodynamics 69–71, 871–882.

Sankaran, R., Jancauskas, E.D., 1992. Direct measurement of the aerodynamic admittance of two-dimensional rectangular cylinders in smooth and turbulent flows. Journal of Wind Engineering and Industrial Aerodynamics 41–44, 601–611.

Scanlan, R.H., 1984. Role of indicial functions in buffeting analysis of bridges. ASCE Journal of Structural Engineering 110, 1433–1446.

Scanlan, R.H., 1993. Problematics in formulation of wind-force models for bridge decks. ASCE Journal of Engineering Mechanics 19, 1353–1375.

Scanlan, R.H., 2000. Motion related body-force functions in two-dimensional low-speed flow. Journal of Fluids and Structures 14, 49–63.

Scanlan, R.H., 2001. Reexamination of sectional aerodynamic force functions for bridges. Journal of Wind Engineering and Industrial Aerodynamics 89 (14–15), 1257–1266.

Scanlan, R.H., Jones, N.P., 1990. A minimum design methodology for evaluating bridge flutter and buffeting response. Journal of Wind Engineering and Industrial Aerodynamics 36, 1341–1353.

Scanlan, R.H., Jones, N.P., 1999. A form of aerodynamic admittance for use in bridge aeroelastic analysis. Journal of Fluids and Structures 13, 1017–1027.

Scanlan, R.H., Tomko, J.J., 1971. Airfoil and bridge deck flutter derivatives. ASCE Journal of Engineering Mechanics 97, 1717–1737.

Sears, W.R., 1941. Some aspects of non-stationary airfoil theory and its practical application. Journal of the Aeronautical Sciences 18, 104–108.

Simiu, E., Scanlan, R.H., 1996. Wind Effects on Structures. Wiley, New York.

Theodorsen, T., 1935. General theory of aerodynamic instability and the mechanism of flutter. NACA Technical Report 496, 413–433.

Tubino, F., 2005. Relationships among aerodynamic admittance functions, flutter derivatives and static coefficients for long-span bridges. Journal of Wind Engineering and Industrial Aerodynamics 93, 929–950.

Wagner, H., 1925. Ueber die entstehung des dynamischen auftriebes von tragfluegeln. ZAMM—Journal of Applied Mathematics and Mechanics/Zeitschrift für angewandte Mathematik und Mechanik 5, 17–35.